Joint microseismic quasi-P and SH traveltime inversion for updating VTI parameters

Song Han, Wei Zhang, Jie Zhang, University of Science and Technology of China (USTC)*

Summary

Quantitative measurements of microseismic anisotropy can provide a better understanding of the subsurface and more accurate event locations. In this study, we present a new method directly inverting Thomsen parameters of vertically layered transversely isotropic media with known event locations but unknown origin times. Thus, it could be applied to process perforation shots or microseismic events with locations determined. We assume that the vertical P and SH velocities can be obtained from sonic logging data, but three anisotropic parameters are unavailable. A horizontal slowness shooting method is applied for anisotropic raytracing. The advantages include: Snell's law is included implicitly, and the convexity of the slowness surface guarantees the convergence of the shooting. The inversion scheme is also designed using slowness components as the independent variables, thus the derivations are simple and easy to implement. Finally we test synthetic data to demonstrate the feasibility of the method.

Introduction

Microseismic monitoring is a rapidly growing technology for the need of hydraulic fracturing (Rutledge and Phillips, 2003; Maxwell et al., 2010; Zimmer, 2011). Locating microseismic events is still a challenge due to inaccurate velocity models. Evidence suggests that the oil/gas shale contains a strong anisotropy (Eisner et al., 2011). A common anisotropic phenomenon is the shear wave splitting. SV and SH waves have different propagating speeds that lead to different first arrivals at seismic receivers. Perforation shots are often used to calibrate velocity models. In this study, we intend to invert perforation shots or microseismic events with known locations.

In this study, we first introduce a horizontal slowness shooting method for calculating quasi-P and SH traveltimes of 1-D layered VTI media. In this raytracing method Snell's law is implicitly included, and the convexity of the slowness surfaces guarantees that the shooting procedure can always come to a convergence. Then the detailed inversion algorithm is presented based on slowness variables. Compared with angle based methods, the equations are simple and easy to implement.

A horizontal slowness shooting method

For general VTI media, the slowness surfaces of quasi-P and SH are described by the following equations:

$$S(p_x, p_z) = p_z{}^2 - \frac{B - \sqrt{B^2 - 4C}}{2} \qquad (1)$$

Where

$$B = \frac{1}{\alpha_0^2} + \frac{1}{\beta_0^2} - 2(1 + \delta + (\varepsilon - \delta)\frac{\alpha_0^2}{\beta_0^2})p_x{}^2$$

$$C = ((1 + 2\varepsilon)p_x{}^2 - \frac{1}{\alpha_0^2})(p_x{}^2 - \frac{1}{\beta_0^2})$$

and

$$S(p_x, p_z) = p_z^2 + (1 + 2\gamma)p_x^2 - \frac{1}{\beta_0^2} \qquad (2)$$

The five Thomsen parameters are α_0 β_0 ε δ and γ . p_x and p_z represent horizontal and vertical slowness components. Actually both these two slowness surfaces are extracted from Christoffel matrix under the existence principle. If the slowness surfaces equal to zero, they are the same as the common phase velocities expressed in the phase angle form. Since both the raytracing and inversion algorithm are based on slowness components, we prefer the slowness surfaces rather than the general phase velocities.

Before starting the shooting method, we should obtain the shooting range. We first compare the maximal horizontal slowness components of the layers between the source and the receiver, and the minimal one is the shooting upper boundary. Obviously zero is the lower boundary. The determination of the shooting range avoids refractions, and ensures that all the calculated traveltimes are related to the direct wave.

One key problem in most shooting methods is the shooting shadows. Before explaining this problem, we first introduce group velocity vectors in slowness domain. Typical slowness surfaces of quasi-P and SH are drawn in Figure 1. The group velocity vector is normal to the tangent line of the point on the slowness surface, just like point A shown in Figure 1.

Kim (2000) proves the local convexity of quasi-P slowness surface. For quasi-SH, the slowness surface is elliptical, obviously it is also convex. If the point on the slowness surface moves slowly from B to C in Figure 1, the corresponding group velocity vector changes smoothly from vertical direction to horizontal. Due to the convexity, this group velocity vector changing is continuous. In our raytracing method, what we shoot is just the horizontal coordinate of this point. In another word, our shooting method covers all the directions and has no shadow zone.

Although we analyze the problem in a homogeneous model, the conclusion is also valid in layered model. Because all the layer slowness surfaces are convex, and all the group velocity vector changings are continues.

Snell's law in layered anisotropic media suggests that the horizontal slowness component is continuous. It means that once a horizontal slowness component is given, it should be constant in all the layers. This allows us to calculate group velocities in different layers simultaneously without any interface processing. Equation (3) is used to calculate group velocities, when the slowness components are known.

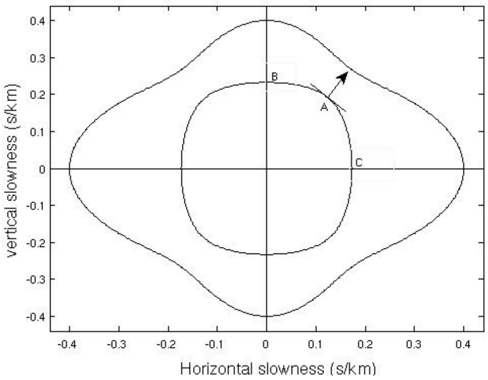

Figure 1: The inner curve is quasi-P slowness surface, and the outer is quasi-SV. Point A is on the surface, the arrow represents group velocity vector.

$$v_g^x = (p_x \frac{\partial S(p_x,p_z)}{\partial p_x} + p_z \frac{\partial S(p_x,p_z)}{\partial p_z})^{-1} \frac{\partial S(p_x,p_z)}{\partial p_x}$$
$$v_g^z = (p_x \frac{\partial S(p_x,p_z)}{\partial p_x} + p_z \frac{\partial S(p_x,p_z)}{\partial p_z})^{-1} \frac{\partial S(p_x,p_z)}{\partial p_z} \quad (3)$$

Where v_g^x and v_g^z are horizontal and vertical group velocity components. $S(p_x, p_z)$ is either quasi-P or SH slowness surface.

Here we summarize the shooting workflow.

(1) Compare the maximal p_x from source layer to receiver and determine the shooting range.
(2) Select the middle p_x of the range, and calculate the group velocities.
(3) Compute raypath and narrow the shooting range according to whether the raypath is above or below the receiver.
(4) If the shooting range is small enough, output the traveltime and raypath, else repeat step (2) and (3).
(5) Go to the next receiver and repeat step (1) to (4).

Many tests show that the vertical distance measured from the receiver to the raypath can be as least as 0.001m. Thus our shooting method is acceptable.

Inversion algorithm

The sensitivity of traveltime T with respect to the Thomsen parameters in layer k is as follows:

$$\frac{\partial T^k}{dm_k} = \frac{\partial(\frac{l_k}{\sqrt{v_x^2+v_z^2}})}{dm_k} = -\frac{l_k}{v_x^2+v_z^2}\frac{1}{\sqrt{v_x^2+v_z^2}}(v_x\frac{\partial v_x}{dm_k}+v_z\frac{\partial v_z}{dm_k}) \quad (4)$$

Where l_k is the length of ray path in layer k. v_x and v_z are horizontal and vertical components of group velocity. m_k is one of the Thomsen parameters except for α_0 and β_0. We can see the only unknown terms are the total differentials of group velocity components with respect to m_k. Here we expand these terms:

$$\frac{\partial v_x}{dm_k} = \frac{\partial v_x}{\partial p_z}(\frac{\partial p_z}{\partial p_x}\frac{\partial p_x}{\partial m_k}+\frac{\partial p_z}{\partial m_k}) + \frac{\partial v_x}{\partial p_x}\frac{\partial p_x}{\partial m_k} + \frac{\partial v_x}{\partial m_k}$$
$$\frac{\partial v_z}{dm_k} = \frac{\partial v_z}{\partial p_z}(\frac{\partial p_z}{\partial p_x}\frac{\partial p_x}{\partial m_k}+\frac{\partial p_z}{\partial m_k}) + \frac{\partial v_z}{\partial p_x}\frac{\partial p_x}{\partial m_k} + \frac{\partial v_z}{\partial m_k} \quad (5)$$

p_z can be expressed by p_x through the slowness surface, thus the derivative of p_z with respect to p_x can be obtained. The expression of group velocity components is known, and therefore all the terms in Equation (5) can be determined except for the derivative of p_x with respect to m_k. To obtain this term, an assumption is made that the raypath perturbation is of higher order to the traveltime perturbation (li et al., 2013), and it writes as:

$$\frac{\partial \theta_g}{dC_{ij}} = 0 \quad (6)$$

Where θ_g is the group velocity angle, C_{ij} is the stiffness component. We rewrite it as:

$$\frac{\partial \arctan(v_z/v_x)}{dm_k} = \frac{1}{\sqrt{1+(v_z/v_x)^2}}\frac{\partial(v_z/v_x)}{dm_k}$$
$$= \frac{1}{\sqrt{1+(v_z/v_x)^2}}\frac{v_x\frac{\partial v_z}{dm_k} - v_z\frac{\partial v_x}{dm_k}}{v_x^2} = 0 \quad (7)$$

Obviously the numerator should equal to zero. Thus,

$$v_x \frac{\partial v_z}{dm_k} - v_z \frac{\partial v_x}{dm_k} = 0 \tag{8}$$

Substitute Equation (5) into (8), we achieve:

$$\frac{\partial p_x}{\partial m_k} = \frac{\frac{v_z}{2p_z}\frac{\partial v_x}{\partial p_z}\frac{\partial g(p_x)}{\partial m_k} + v_z\frac{\partial v_x}{\partial m_k} - \frac{v_x}{2p_z}\frac{\partial v_z}{\partial p_z}\frac{\partial g(p_x)}{\partial m_k} - v_x\frac{\partial v_z}{\partial m_k}}{\frac{v_x}{2p_z}\frac{\partial v_z}{\partial p_z}\frac{\partial g(p_x)}{\partial p_x} + v_x\frac{\partial v_z}{\partial p_x} - \frac{v_z}{2p_z}\frac{\partial v_x}{\partial p_z}\frac{\partial g(p_x)}{\partial p_x} - v_z\frac{\partial v_x}{\partial p_x}} \tag{9}$$

Solving Equation (9), we finally obtain the derivative of p_x with respect to m_k. For origin time, the sensitivity is simply equal to one. The scheme used for the inversion is set as:

$$A\begin{pmatrix}\Delta m \\ \Delta t_0\end{pmatrix} = \Delta d \tag{10}$$

where A is the sensitivity matrix which includes two parts. One is the sensitivities of Thomsen parameters, and the other one is of the origin times. Δm is the model perturbation. Δt_0 is the origin time perturbation, and Δd is the traveltime residual. A SVD method is applied to solve Equation (10). We iteratively do the inversion until the L2-norm misfit is stable.

Numerical examples

We first conduct a simple synthetic example with known α_0 and β_0. The relative geometry is shown in Figure 1. The color bar represents the value of α_0.

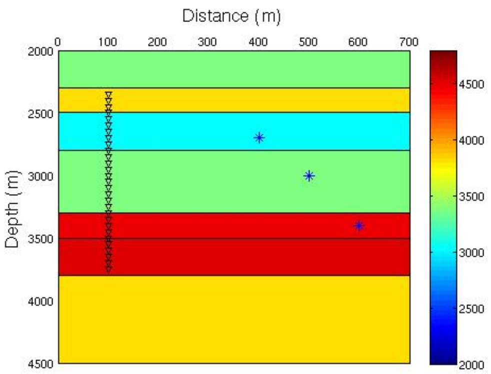

Figure 1: The blue stars are events, and the black triangles represent receivers.

There are twenty nine receivers in well and three events. The receivers locate in a vertical well starting from the depth of 2350m to 3750m and the interval is 50m. The events are placed in different layers with event 1 (400m, 2700m), event 2 (500m, 3000m) and event 3 (600m, 3400m). To test how well our method can invert the origin times, we add different time delays to the recorded traveltimes with Event 1 0.2s, event 2 0.3s and event 3 0.4s. In our program, there is no need to provide any origin times, they are estimated roughly according to the traveltimes curves, and then updated during the inversion. The final results are shown from Figure 2 to Figure 4.

We can see that all the parameters are inverted well except for the first and last layers, which have no rays passing through. In Table 1, the inverted origin time information is provided.

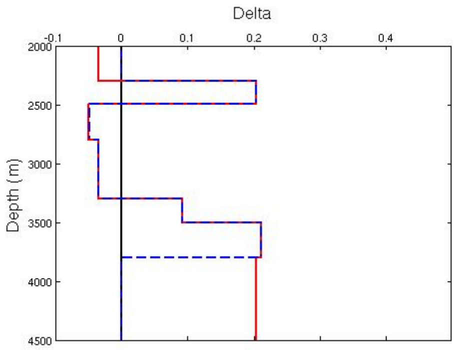

Figure 2: The black line is the initial value of δ. Red line is the true value and the blue dash line is the inverted value.

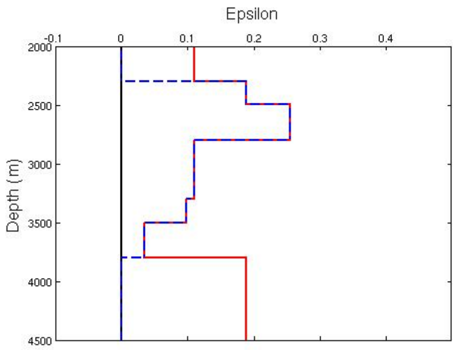

Figure 3: The black line is the initial value of ε, red line is the true value and the blue dash line is the inverted value.

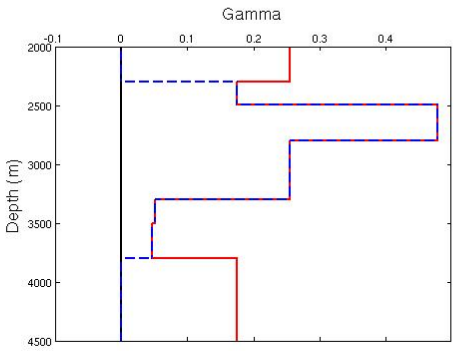

Figure 4: The black line is the initial value of γ , red line is the true value and the blue dash line is the inverted value.

Event	Initial t0 (s)	Final t0 (s)	True t0 (s)
1	1.86221e-1	2.00002e-01	2.0e-01
2	2.82504e-1	3.00003e-01	3.0e-01
3	3.84981e-1	4.00004e-01	4.0e-01

Table 1: Initial, finial and true origin times for each event.

We perform 400 iterations to achieve the final results. However it only costs 4sec.

In the second test, we add five percent perturbations to α_0 and β_0 in the third layer. For VTI media, ε and γ are related to the ratio of horizontal and the vertical velocities of quasi-P and SH. Generally if α_0 and β_0 are smaller than the true values, the inverted ε and γ should increase to make a compensation for the traveltimes. However from Figure 5, we obtain the opposite conclusion, this is caused by the trade-off of origin time. The inverted origin time information is shown in the Table 2.

Event	Final t0(s) +5 percent	Final t0(s) -5 percent	True t0 (s)
1	2.14857e-1	1.82614e-01	2.0e-01
2	3.18028e-1	2.80428e-01	3.0e-01
3	4.23123e-1	3.74827e-01	4.0e-01

Table 2: Second column is the inverted origin times with five percent perturbations added, and the third with negative five percent perturbations. The final column is the true origin times.

A clear trade-off between the origin time and anisotropic parameters can be seen from this test. Even though the trade-off exits, the shapes of final anisotropic parameter curve are similar to the true ones. For real case if the logging data is not accurate enough, α_0 and β_0 should be involved in the inversion.

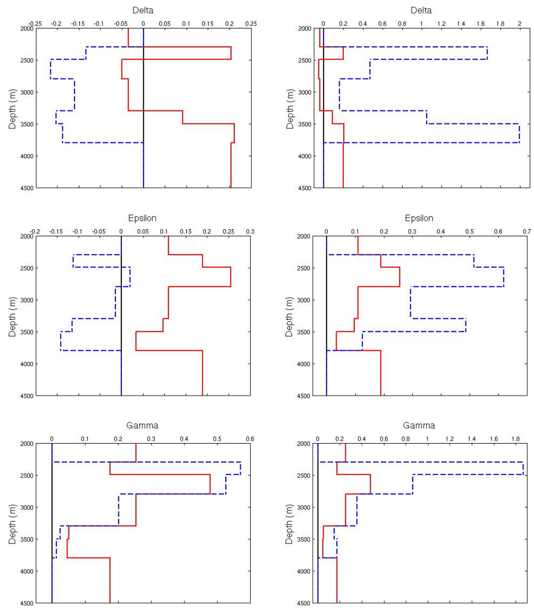

Figure 5: The black line is the initial value. Red line is the true value and the blue dash line is the inverted value. Five positive present perturbations is added to the α_0 and β_0 of the left column, and negative five percent to the right column.

Conclusions

We develop a joint traveltime inversion method for layered VTI media with unknown origin time. Both raytracing and inversion algorithm are derived based on slowness variables. Thus the derivations are simple and clear. The convexity of slowness surface guarantees the convergence of our shooting algorithm and the implementation of the inversion is easy. The synthetic test demonstrates the feasibility and validity of our method. However if α_0 and β_0 extracted for the logging data are not accurate enough, the trade-off between anisotropic parameters and origin time should be a concern.

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