Cross double-difference inversion method for microseismic location

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Summary

The double-difference (DD) location method has long been applied for locating a cluster of earthquakes, with data recorded at surface seismic stations. We extend the approach for locating a cluster of microseismic events with data recorded from a single monitoring well. Synthetic tests indicate that the DD method for the downhole microseismic monitoring is sensitive to picking errors, and results are reasonable in relative event locations, but poor in absolute locations. We modify the approach by using cross traveltime difference between P-wave arrival of one event and S-wave arrival of another event for inversion instead of traveltime differences of the same phase. Our simulation indicates that this Cross Double-Difference (CDD) method can reduce the dependence on the initial guess for event locations and also improve accuracy of microseismic event cluster in both relative and absolute locations. This is because the cross double-difference quantity is much larger than the double difference of the same phase and less affected by picking errors.

Introduction

Microseismic monitoring, since the initial idea in the 1970s and its commercialization around 2000, has proven to be a vital tool for understanding underground process during hydraulic fracturing. The microseismic events induced by hydraulic fracturing can be detected by multiple receivers deployed in monitoring wells or on the surface (Warpinski et al., 2009). In a production field, we can apply microseismic information to understand fracture growth (Cipolla, et al., 2011). Therefore, it is important to infer accurate locations of microseismic events. Current microseismic location methods include, for example, grid search, double-difference, migration-based imaging, and search engine methods (Rodi et al., 2006; Reshetnikov et al., 2010; Zhang et al., 2014). Several factors affect the accuracy of event locations; such as recording geometry, uncertainty in the velocity model, and traveltime picking errors (Castellanos et al., 2013).

The double-difference (DD) location method is a relative location approach and it was developed to reduce the effect of an inaccurate velocity model in seismological applications (Waldhauser and Ellsworth, 2000). The approach requires arrival time picks from multiple events and infers their relative locations. Zhang et al. (2006) developed a double-difference tomography method (TomoDD) using both absolute and relative arrival times for surface recorded data. It inverts the event locations and the velocity structure simultaneously. Zhang et al. (2009) proposed using the differential S-P times to remove the effect of different raypaths of P- and S-waves outside the source region. Rudziński et al. (2013) developed the extended double difference (EDD) method by considering all possible combinations of source-station pairs and focus on the reliability of source depth estimation for mining-induced seismic events.

Recently, the DD location method has been applied to determine microseismic locations (Castellanos et al., 2013; Li et al., 2013). It can help to obtain accurate relative event locations, which are crucial for fracture analysis in microseismic monitoring. For microseismic monitoring, the distance between two events is small, the traveltime difference of P- and S-wave will be affected by picking errors easily. So the absolute location results are dependent on the initial locations in inversions. Considering this deficiency of DD method, we propose a new location method which can provide both relative and absolute locations. We call it the Cross Double-Difference method (CDD). It uses the arrival time difference of P1-S2 and P2-S1 instead of P1-P2 and S1-S2. It provides large data magnitude than DD method and the results are less dependent on the initial locations.

Double-difference and cross double-difference theory

The double-difference method is an earthquake location method, and it is developed for obtaining the relative location of hypocenters without the need of a sufficient knowledge of the earth model (Waldhauser and Ellsworth, 2000). If we consider both P- and S-wave, the objective function $\emptyset$ for the DD location method can be written as:

$$\emptyset = \sum_{k=1}^{mr} \sum_{i=1}^{ns-1} \sum_{j=i+1}^{ns} \left\{ \left(T_{kp}^{i} - T_{kp}^{j}\right)^{obs} - \left(T_{kp}^{i} - T_{kp}^{j}\right)^{cal} + \left(T_{ks}^{i} - T_{ks}^{j}\right)^{obs} - \left(T_{ks}^{i} - T_{ks}^{j}\right)^{cal} \right\}, \quad (1)$$

where T_{kp}^{i} is absolute arrival time of P-wave from event *i* to receiver *k*, including the traveltime and origin time. $\left(T_{kp}^{i} - T_{kp}^{j}\right)^{obs}$ is the observed arrival time difference between event pairs and $\left(T_{kp}^{i} - T_{kp}^{j}\right)^{cal}$ is the theoretical arrival time difference calculated by initial locations. Here ns is the source number and mr is receiver number. From the theory of double-difference method, the relationship between event location and traveltime is highly nonlinear. After using a truncated Taylor expansion to linearize this relationship, the residuals are linearly related to the parameter perturbations. From event *i* to receiver *k*, we can obtain the equation 2:

$$\frac{\partial t_k^i}{\partial \mathbf{m}} \Delta \mathbf{m}^i = \boldsymbol{r}_k^i, \qquad (2)$$

where $\boldsymbol{r}_k^i = (t^{obs} - t^{cal})_k^i$. $\Delta \mathbf{m}^i$ is the parameter perturbation, including the location ($\Delta x^i, \Delta y^i, \Delta z^i$) and the origin time ($\Delta \tau^i$). We can get double-difference equation with P- and S-wave by subtracting a similar equation for event j to receiver k:

$$d\boldsymbol{r}_{kP}^{i,j} = \boldsymbol{r}_{kP}^i - \boldsymbol{r}_{kP}^j = \frac{\partial t_{kP}^i}{\partial \boldsymbol{m}} \Delta \boldsymbol{m}^i - \frac{\partial t_{kP}^j}{\partial \boldsymbol{m}} \Delta \boldsymbol{m}^j$$

$$d\boldsymbol{r}_{kS}^{i,j} = \boldsymbol{r}_{kS}^i - \boldsymbol{r}_{kS}^j = \frac{\partial t_{kS}^i}{\partial \boldsymbol{m}} \Delta \boldsymbol{m}^i - \frac{\partial t_{kS}^j}{\partial \boldsymbol{m}} \Delta \boldsymbol{m}^j. \qquad (3)$$

We can combine all event pairs for all receivers to form a system of linear equations of the following form：

$$\mathbf{G} \Delta \mathbf{m} = \Delta \mathbf{d}, \qquad (4)$$

where G is the sensitivity matrix containing the partial derivatives of differential arrival times (P- and S-wave) with respect to the model parameters. We can use singular value decomposition (SVD) method or conjugate gradient (CG) method to solve this inversion equation.

Based on equation 1, we improve the objective function $\emptyset$,

$$\emptyset = \sum_{k=1}^{mr} \sum_{i=1}^{ns-1} \sum_{j=i+1}^{ns} \Big\{ \big(T_{kp}^i - T_{ks}^j\big)^{obs} - \big(T_{kp}^i - T_{ks}^j\big)^{cal} + \big(T_{kp}^j - T_{ks}^i\big)^{obs} - \big(T_{kp}^j - T_{ks}^i\big)^{cal} \Big\}, \qquad (5)$$

We still use the differential arrival times between two events. The magnitude of the observed data is improved since we use arrival time difference of P1-S2 and P2-S1 instead of P1-P2 and S1-S2, so we call it the Cross Double-Difference (CDD) method. The relative information is reserved and the magnitude of observed data is similar with the S-P traveltime difference of individual event. As a consequence, we can obtain relative and absolute locations simultaneously.

Existing problems of the double-difference method

Majority of microseismic events have low signal-to-noise ratio (S/N), and the associated impact of this low S/N is the significant challenge in data processing (Maxwell et al. 2010). Figure 1 shows the geometry of our monitoring system with a single well. We design a horizontal layer model. There are 10 receivers in the borehole with 15 m interval, and 4 microseismic event coordinates are shown in Table 1.

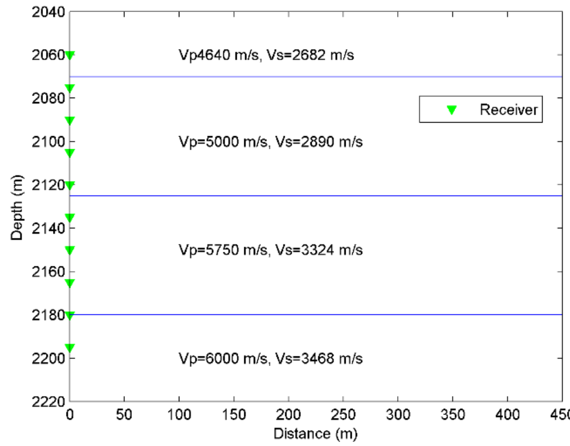


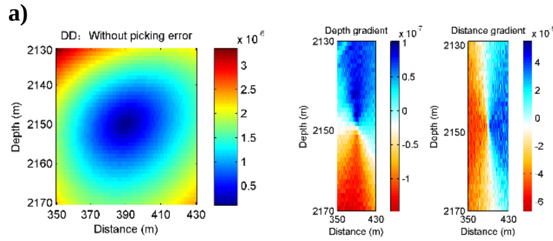
Figure 1: Velocity model and acquisition geometry with 10 receivers (green triangles) in a vertical monitoring well.

Table 1: True event locations and searching ranges of Figure 2.

Event ID (X , Z) (m)	X range (m)	Z range (m)
Event 1 (390,2150)	350 to 430	2130 to 2170
Event 2 (390,2151)	350 to 430	2131 to 2171
Event 3 (390,2152)	350 to 430	2132 to 2172
Event 4 (390,2153)	350 to 430	2133 to 2173

In field data, there are some picking errors when we pick the waveforms. We add 0.3 ms random noise to simulate the picking errors. Figure 2 shows the traveltime misfit distribution of different inversion methods, and we make a comparison on the data with no picking errors and with 0.3 ms picking errors. The coordinate of true locations and searching ranges are listed in Table 1. DD method does not have local minimum and its global minimum is close to the true locations when picking errors are free. When we add 0.3 ms noise to the arrival times, the misfit plot is filled with local minimum, which the gradient along different directions can illustrate this problem clearly (Figure 2a-b). Under this circumstance, the final absolute locations are totally dependent on the initial locations. That is why DD method is only a relative relocation method.

a)



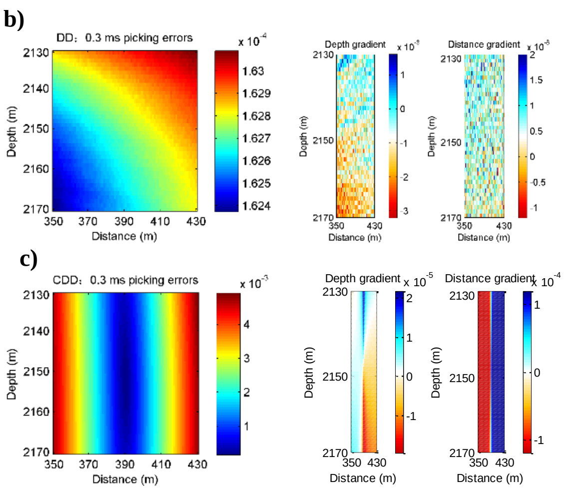


Figure 2: Distribution of traveltime residuals and gradient in both depth and distance direction. (a) DD method without picking errors. (b) DD method with 0.3ms picking errors. (c) CDD method with 0.3ms picking errors.

Next, we analyze the magnitude of the observed data (Figure 3). We plot the traveltime difference of P- and S-wave calculated between event pairs. In Figure 3, the observed data magnitude of DD method is about 6 ms, while the magnitude of CDD method is about 40-60 ms. The sampling rate in the microseismic records about 0.25 ms ~ 0.5 ms. Thus, we cannot get higher precision in filed data.

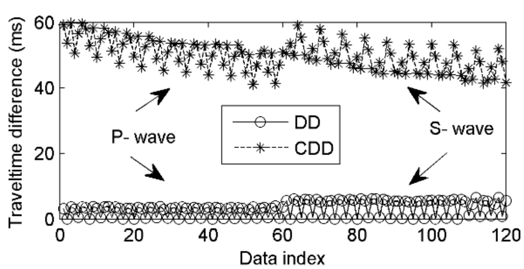


Figure 3: The magnitude of the observed traveltime difference of DD and CDD method. The circles are DD traveltime difference and stars are CDD traveltime difference.

In order to obtain accurate event locations, we suggest to use the large CDD quantity for inversion. We plot the traveltime misfit distribution with 0.3 ms noise to verify that the CDD method is independent of the initial locations (see Figure 2b-c). It shows that CDD method does not have local minimum and its global minimum is around the true locations. Therefore, we can get accurate absolute locations no matter where the initial locations are. Besides, the observed data magnitude is about 50 ms (Figure 3). It further shows that the CDD method has higher magnitude data than DD method, therefore it can obtain the absolute locations even with some picking errors.

Numerical examples

Here we show an example for the microseismic relocation with a set of microseismic events (20 events) and 10 receivers in the downhole monitoring well. Results show that no matter where the initial locations are, the DD method is able to get the true locations when there is no picking errors. Since the microseismic data has a low SNR, we cannot avoid some picking errors when we process the data.

Figure 4 indicates that DD method could not resolve the absolute locations when we add 0.3 ms noise. The finial absolute inversion results completely depend on the barycenter of the initial event locations. But the relative locations are still in a fracture. We can get better absolute location results when the barycenter is close to the true locations. When we apply the CDD method to relocate these events, we can get correct results on both the absolute and relative locations no matter where the initial locations are, even with noise added (Figure 4a-c).

Conclusions

In this study, we develop a cross double-difference location method which can provide us accurate results on both absolute and relative locations. Through the theoretical analysis, DD method has uniqueness solution and it is able to obtain the absolute locations. However, the inversion results will rely on the barycenter of initial locations if there is picking errors. Sampling rate will also have an impact on the finial locations because of the small data magnitude. In order to prove that our CDD method can provide accurate absolute locations, we make synthetic test and compare two methods on traveltime misfit distribution and data magnitude. Numerical experiments indicate that the CDD method can provide better location results on both relative and absolute locations than DD method.

Acknowledgements

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a)

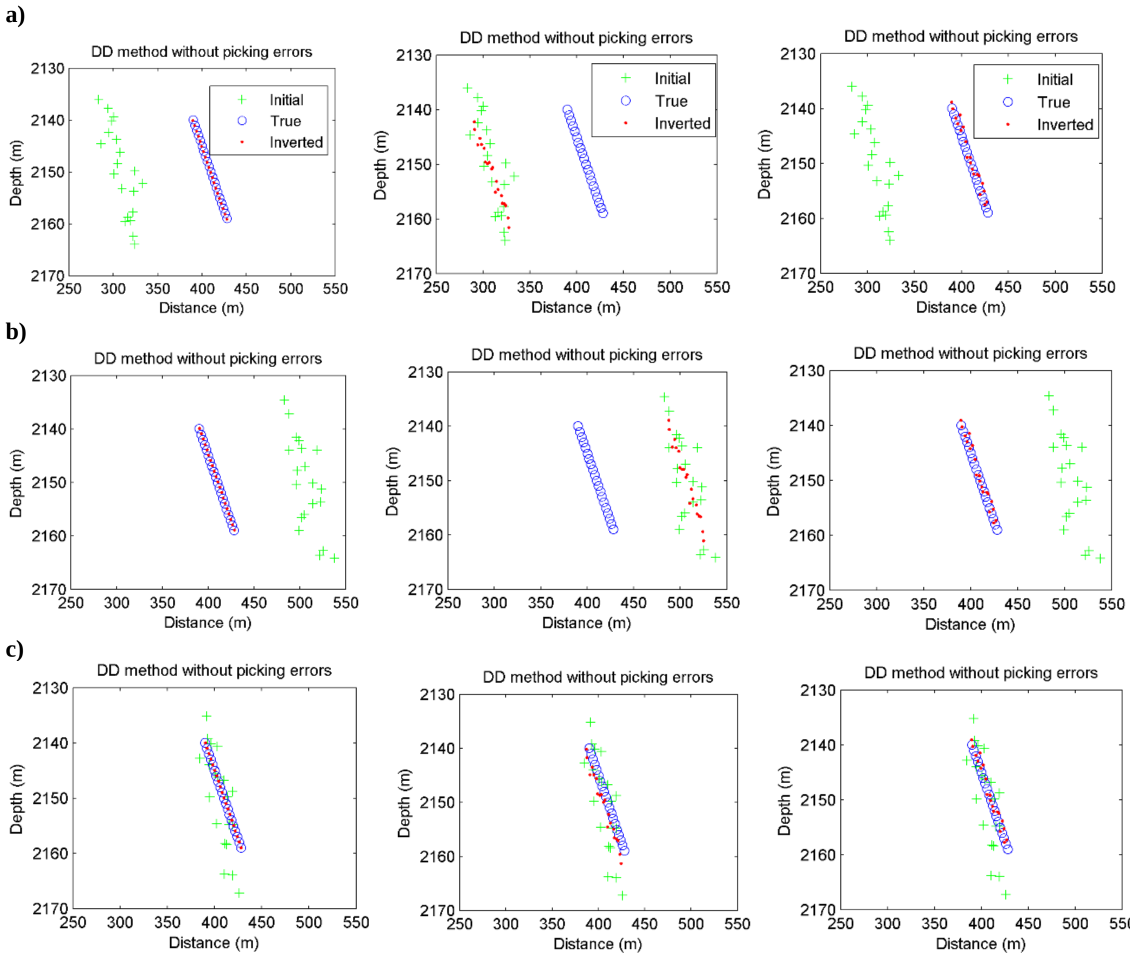

Figure 4: Location results of DD and CDD method with the initial locations on the left, right and near the true locations, respectively. (a) DD method with no picking errors. (b) DD method with 0.3 ms picking errors. (c) CDD method with 0.3 ms picking errors.